2-2 Free Fall

Vocabulary **Free Fall:** The movement of an object in response to a gravitational attraction.

When an object is released, it falls toward Earth due to the gravitational attraction Earth provides. As the object falls, it will accelerate at a constant rate of 9.8 m/s^2 regardless of its mass. However, to make calculations more expedient and easier to do without a calculator, this number will be written as $g = 10.0\ m/s^2$ throughout this book.

There are many different ways to solve free fall exercises. The sign convention used may be chosen by you or your teacher. In this book, the downward direction will be positive, and anything falling downward will be written with a positive velocity and position; anything moving upward will be represented with a negative velocity and position. Remember: Gravity *always* acts to pull an object down, so the gravitational acceleration, g, will always be written as a positive number regardless of which direction the object is moving.

The displacement of a falling object in a given amount of time is written as

$$\Delta d = v_o \Delta t + \left(\frac{1}{2}\right) g \Delta t^2$$

The final velocity of a falling object can be represented by the equation

$$v_f^2 = v_o^2 + 2g\Delta d$$

or by the earlier equation, $a = (v_f - v_o)/\Delta t$, rewritten as $v_f = v_o + a\Delta t$, or

$$v_f = v_o + g\Delta t$$

Note that the term "g" in all of these exercises can be written as "a" if you use a constant acceleration other than gravity. Therefore, these equations can be used for objects moving horizontally as well as vertically.

It is common to neglect air resistance in most free fall exercises (including those in this book), although in real life, air resistance is a factor that must be taken into account. This book will also assume that the initial speed of all objects in free fall is zero, unless otherwise specified.

Solved Examples

Example 6: King Kong carries Fay Wray up the 321-m-tall Empire State Building. At the top of the skyscraper, Fay Wray's shoe falls from her foot. How fast will the shoe be moving when it hits the ground?

Given: $v_o = 0$ m/s
$g = 10.0$ m/s^2
$\Delta d = 321$ m

Unknown: $v_f = ?$
Original equation: $v_f^2 = v_o^2 + 2g\Delta d$

Solve: $v_f = \sqrt{v_o^2 + 2g\Delta d} = \sqrt{0 + 2(10.0 \text{ m/s}^2)(321 \text{ m})} = \sqrt{6420 \text{ m}^2/\text{s}^2}$
$= \mathbf{80.1 \text{ m/s}}$

Example 7: The Steamboat Geyser in Yellowstone National Park, Wyoming, is capable of shooting its hot water up from the ground with a speed of 48.0 m/s. How high can this geyser shoot?

Solution: Remember, the geyser is shooting **up;** therefore it must have a negative initial velocity.

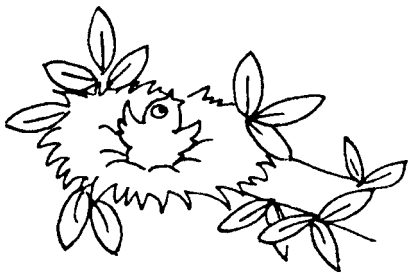
Given: $v_o = -48.0$ m/s
$v_f = 0$ m/s
$g = 10.0$ m/s^2

Unknown: $\Delta d = ?$
Original equation: $v_f^2 = v_o^2 + 2g\Delta d$

Solve: $\Delta d = \dfrac{v_f^2 - v_o^2}{2g} = \dfrac{(0 \text{ m/s})^2 - (-48.0 \text{ m/s})^2}{2(10.0 \text{ m/s}^2)} = \mathbf{-115 \text{ m}}$

As you might expect, the final answer has a negative displacement. This means that the total distance the water has traveled is measured up from the ground.

Example 8: A baby blue jay sits in a tall tree awaiting the arrival of its dinner. As the mother lands on the nest, she drops a worm toward the hungry chick's mouth, but the worm misses and falls from the nest to the ground in 1.50 s. How high up is the nest?



Given: $v_o = 0$ m/s
$g = 10.0$ m/s^2
$t = 1.50$ s

Unknown: $\Delta d = ?$
Original equation: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2 = 0 + \left(\frac{1}{2}\right)(10.0 \text{ m/s}^2)(1.50 \text{ s})^2 = \mathbf{11.3 \text{ m}}$

Example 9: A giraffe, who stands 6.00 m tall, bites a branch off a tree to chew on the leaves, and he lets the branch fall to the ground. How long does it take the branch to hit the ground?

Given: $\Delta d = 6.00$ m
$g = 10.0$ m/s^2
$v_o = 0$ m/s

Unknown: $\Delta t = ?$
Original equation: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $\Delta t = \sqrt{\dfrac{2\Delta d}{g}} = \sqrt{\dfrac{2(6.00 \text{ m})}{10.0 \text{ m/s}^2}} = \sqrt{1.20 \text{ s}^2} = \mathbf{1.10 \text{ s}}$

Practice Exercises

Exercise 9: Billy, a mountain goat, is rock climbing on his favorite slope one sunny spring morning when a rock comes hurtling toward him from a ledge 40.0 m above. Billy ducks and avoids injury. a) How fast is the rock traveling when it passes Billy? b) How does this speed compare to that of a car traveling down the highway at the speed limit of 25 m/s (equivalent to 55 mi/h)?

a) $v_f^2 = v_o^2 + 2g\Delta d = 0 + 2(10.0\ \text{m/s}^2)(40.0\ \text{m}) = 800.\ \text{m}^2/\text{s}^2$
$v_f = \sqrt{800.\ \text{m}^2/\text{s}^2} =$ **28.3 m/s**
b) A little faster than a car moving at 25 m/s.

Answer: **a.** **28.3 m/s**

Answer: **b.** **slightly faster**

Exercise 10: Reverend Northwick climbs to the church belfry one morning to determine the height of the church. From an outside balcony he drops a book and observes that it takes 2.00 s to strike the ground below. a) How high is the balcony of the church belfry? b) Why would it be difficult to determine the height of the belfry balcony if the Reverend dropped only one page out of the book?

a) $\Delta d = v_o\Delta t + (1/2)g\Delta t^2 = 0 + (1/2)(10.0\ \text{m/s}^2)(2.00\ \text{s})^2 =$ **20.0 m**
b) Air resistance would cause the paper to float instead of drop.

Answer: **a.** **20.0 m**

Answer: **b.** **The paper would float.**

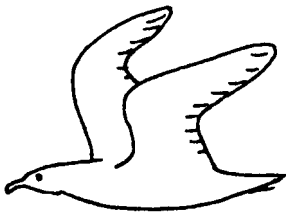
Exercise 11: How long is Tina, a ballerina, in the air when she leaps straight up with a speed of 1.8 m/s?

$\Delta t = (v_f - v_o)/g = [0\ \text{m/s} - (-1.8\ \text{m/s})]/(10.0\ \text{m/s}^2) = 0.18\ \text{s}$
Total time is $2(0.18\ \text{s}) =$ **0.36 s.**

Answer: **0.36 s**

Exercise 12: In order to open the clam it catches, a seagull will drop the clam repeatedly onto a hard surface from high in the air until the shell cracks. If a seagull flies to a height of 25 m, how long will the clam take to fall?

$\Delta t = \sqrt{2\Delta d/g} = \sqrt{2(25\text{ m})/(10.0\text{ m/s}^2)} =$ **2.2 s**



Answer: **2.2 s**

Exercise 13: At an amusement park, a popular ride known as "Free Fall" carries passengers up to a height of 33.5 m and drops them to the ground inside a small cage. How fast are the passengers going at the bottom of this exhilarating journey?

$v_f^2 = v_o^2 + 2g\Delta d = 0 + 2(10.\text{ m/s}^2)(33.5\text{ m}) = 670.\text{ m}^2/\text{s}^2$
$v_f = \sqrt{670.\text{ m}^2/\text{s}^2} =$ **25.9 m/s**

Answer: **25.9 m/s**

Exercise 14: A unique type of basketball is played on the planet Zarth. During the game, a player flies above the basket and drops the ball in from a height of 10 m. If the ball takes 5.0 s to fall, find the acceleration due to gravity on Zarth.

$g = 2\Delta d/\Delta t^2 = 2(10\text{ m})/(5.0\text{ s})^2 =$ **0.8 m/s^2**

Answer: **0.8 m/s^2**

Additional Exercises

A-1: During an Apollo moon landing, reflecting panels were placed on the moon. This allowed earth-based astronomers to shoot laser beams at the moon's surface to determine its distance. The reflected laser beam was observed 2.52 s after the laser pulse was sent. If the speed of light is 3.00×10^8 m/s, what was the distance between the astronomers and the moon?

A-2: The peregrine falcon is the world's fastest known bird and has been clocked diving downward toward its prey at constant vertical velocity of 97.2 m/s. If the falcon dives straight down from a height of 100. m, how much time does this give a rabbit below to consider his next move as the falcon begins his descent?

A-3: The Kentucky Derby, the first leg of horse racing's Triple Crown, was won on May 5, 2007 by Street Sense with a time of 122.2 s. If the race covers 2011.25 m, what was Street Sense's average speed a) in m/s? b) in mi/h? (Read more about the Kentucky Derby at http://www.kentuckyderby.com)

A-4: During the gas crisis of the 1970's, the posted highway speed limit was dropped to 88.5 km/h (55 mi/h) across the country. By the 1990s, many stretches of highway had put the posted speed limit back to 104.6 km/h (65 mi/h). In Maine, the distance from Portland to Bangor is 215 km. How much time can be saved in traveling from Portland to Bangor at this higher speed limit?

A-5: A tortoise and a hare are in a road race to defend the honor of their breeds. The tortoise crawls the entire 1000.-m distance at a speed of 0.2000 m/s while the rabbit runs the first 200.0 m at 2.000 m/s. The rabbit then stops to take a nap for 1.300 h and awakens to finish the last 800.0 m with an average speed of 3.000 m/s. a) Who wins the race and by how much time? b) Draw a graph of distance vs. time for the situation.

A-6: Two physics professors challenge each other to a 100.-m race across the football field. The loser will grade the winner's physics labs for one month. Dr. Nelson runs the race in 10.40 s. Dr. Bray runs the first 25.0 m with an average speed of 10.0 m/s, the next 50.0 m with an average speed of 9.50 m/s, and the last 25.0 m with an average speed of 11.1 m/s. Who gets stuck grading physics labs for the next month?

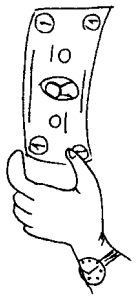
A-7: A caterpillar crawling up a leaf slows from 0.75 cm/s to 0.50 cm/s at a rate of -0.05 cm/s^2. How long does it take the caterpillar to make the change?

A-8: In the *Wizard of Oz,* Dorothy awakens in Munchkinland where her house has been blown by a tornado. If the house fell from a height of 3000. m, with what speed did it hit the Wicked Witch of the East when it landed?

A-9: The Tambora volcano on the island of Sumbawa, Indonesia, has been known to throw ash into the air with a speed of 625 m/s during an eruption. a) How high could this volcanic plume have risen? b) On Venus, where the acceleration due to gravity is slightly less than on Earth, would this volcanic plume rise higher or not as high as it does on Earth?

A-10: Chief Boolie is out hunting when a coconut falls from a tree and lands on his toe. If the nut fell for 1.4 s, how fast was it traveling when it hit Chief Boolie's toe?

A-11: Here is a bet that you are almost sure to win! Try dropping a dollar bill through a friend's fingers and offer to let her keep it if she can catch it. The bill should be started just at the finger level and your friend shouldn't have any advanced warning when it is going to drop. A dollar bill has a length of 15.5 cm and human reaction time is rarely less than 0.20 s. Do the necessary calculations—why is this almost a sure bet?



A-12: While repairing a defective radio transmitter on the roof of the 442-m-tall Sears Tower in Chicago, Lyle drops his hammer, which falls all the way to the ground below. a) How long will it take for Lyle's hammer to fall? b) With what speed will the hammer hit the pavement? c) How far will the hammer have fallen after 1.50 s when a janitor watches it pass outside an office window? (Read more about America's tallest building at http://www.searstower.org)

A-13: On July 31, 1994, Sergei Bubka of the Ukraine broke his own world pole-vaulting record by attaining a height of 6.14 m. a) How long did it take Bubka to return to the ground from the highest part of his vault? b) Describe how this time compares to the time it took him to go from the ground to the highest point. c) The woman's pole-vaulting record was set by USA's Stacy Dragila in 2001, when she vaulted 4.81 m. How fast was Dragila moving when she left the ground?

A-14: A Christmas tree ball will break if dropped on a hardwood floor with a speed of 2.0 m/s or more. Holly is decorating her Christmas tree when her cat, Trickor, taps a ball, causing it to fall 15 cm from a tree branch to the floor. Does the ball break?

A-15: Perhaps sometime in the future, NASA will develop a program to land a human being on Mars. If you were the first Mars explorer and discovered that when you dropped a hammer it took 0.68 s to fall 0.90 m to the ground, what would you calculate for the gravitational acceleration on Mars?

Challenge Exercises for Further Study

B-1: Seth is doing his student driving with the "Give-Me-A-Brake" driving school and is traveling down the interstate with a speed of 9.0 m/s. Mack is driving his "18-wheeler" down the fast lane at 27.0 m/s when he notices Seth 30.0 m ahead of him in the right lane. a) If Mack and Seth maintain their speeds, how far must Mack travel before he catches up to Seth? b) How long will this take?

B-2: On August 21, 2004, at the summer Olympics in Athens, Greece, the U.S. men's swim team attempted to beat the previous world record for the 400-m medley relay of 211.54 s set in 2003. The four U.S. men each swam the 100.0-m leg of the race with the following average speeds: Aaron Peirsol (backstroke) at 1.8709 m/s, Brendan Hansen (breaststroke) at 1.6844 m/s, Ian Crocker (butterfly) at 1.9889 m/s, and Jason Lezak (freestyle) at 2.1017 m/s. a) Did the team break the world record? If so, by how much? If not, by how much did they miss it? b) What was the U.S. team's average speed for the 400.0-m race?

B-3: In 1945, the *Enola Gay,* a B-29 bomber, dropped the atomic bomb from a height of 9450 m over Hiroshima, Japan. If the plane carrying the bomb were traveling with a horizontal velocity of 67.0 m/s, how far horizontally would the bomb have traveled between the point of release and the point where it exploded 513 m above the ground? (To avoid being above the bomb when it exploded, the *Enola Gay* turned sharply away after the bomb's release. Read more about the *Enola Gay* at http://www.theenolagay.com)

B-4: Pepe, the clown, is jumping on a trampoline as Babette, the tightrope walker, above him suddenly loses her balance and falls off the tightrope straight toward Pepe. Pepe has just started upward at 15 m/s when Babette begins to fall. Pepe catches her in midair after 1.0 s. a) How far has Babette fallen when she is caught by Pepe? b) What is Babette's velocity at the time of contact? c) What is Pepe's velocity at the time of contact? d) How far above the trampoline was Babette before she fell?

B-5: Mr. DeFronzo has just learned that he won the Presidential Award for Excellence in Science Teaching. He runs to the open window and throws his red marking pen into the air with an initial upward speed of 5.00 m/s. a) If the window is 12.0 m above the ground, what is the velocity of the pen 1.0 s after it is thrown? b) How far has the pen fallen from its starting position after 2.0 s? c) How long does it take the pen to hit the ground?

B-6: On October 24, 1901, Annie Edson Taylor, a school teacher from Michigan, became the first person to successfully ride over Niagara Falls in a wooden barrel. Assume Annie began her journey at Goat Island, 240. m from the falls, where the water current started her down the Niagara River at 8.00 m/s. During her journey, the current reached 15.0 m/s as it carried Annie over Horseshoe Falls, a drop of 51.0 m. How long was Annie's trip from start to finish?