5 Momentum

5-1 Impulse and Momentum

Vocabulary **Momentum:** A measure of how difficult it is to stop a moving object.

$$\textbf{momentum} = \textbf{(mass)(velocity)} \quad \text{or} \quad \boldsymbol{p} = mv$$

If the momentum of an object is changing, as it is when a force is exerted to start it or stop it, the change in momentum can be found by looking at the change in mass and velocity during the interval.

$$\textbf{change in momentum} = \textbf{change in } [(\textbf{mass})(\textbf{velocity})] \quad \text{or} \quad \Delta\boldsymbol{p} = \Delta(mv)$$

For all the exercises in this book, assume that the mass of the object remains constant, and consider only the change in velocity, Δv, which is equal to $v_f - v_o$. Momentum is a vector quantity. Its direction is in the direction of the object's velocity.

The SI unit for momentum is the **kilogram·meter/second (kg·m/s).**

Vocabulary **Impulse:** The product of the force exerted on an object and the time interval during which it acts.

$$\textbf{impulse} = (\textbf{force})(\textbf{elapsed time}) \quad \text{or} \quad \boldsymbol{J} = \boldsymbol{F}\Delta t$$

The SI unit for impulse is the **newton·second (N·s).**

The impulse given to an object is equal to the change in momentum of the object.

$$\boldsymbol{F}\Delta t = m\boldsymbol{\Delta v}$$

The same change in momentum may be the result of a large force exerted for a short time, or a small force exerted for a long time. In other words, impulse is the thing that you *do,* while change in momentum is the thing that you *see.*

The units for impulse and momentum are equivalent. Remember, $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$. Therefore, $1\ \text{N}\cdot\text{s} = 1\ \text{kg}\cdot\text{m/s}$.

Solved Examples

Example 1: Tiger Woods hits a 0.050-kg golf ball, giving it a speed of 75 m/s. What impulse does he impart to the ball? (Read more about Tiger Woods at http://www.tigerwoods.com)

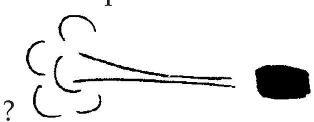
Solution: Because the impulse equals the change in momentum, you can reword this exercise to read, "What was the ball's change in momentum?" It is understood that the ball was initially at rest, so its initial speed was 0 m/s.

Given: $m = 0.050$ kg
$\Delta v = 75$ m/s

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v$

Solve: $\Delta p = (0.050 \text{ kg})(75 \text{ m/s}) = \mathbf{3.8\ kg \cdot m/s}$

Example 2: Wayne hits a stationary 0.12-kg hockey puck with a force that lasts for 1.0×10^{-2} s and makes the puck shoot across the ice with a speed of 20.0 m/s, scoring a goal for the team. With what force did Wayne hit the puck?



Given: $m = 0.12$ kg
$\Delta v = 20.0$ m/s
$\Delta t = 1.0 \times 10^{-2}$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.12 \text{ kg})(20.0 \text{ m/s})}{1.0 \times 10^{-2} \text{ s}} = 240 \text{ kg}\cdot\text{m/s}^2 = \mathbf{240\ N}$

Example 3: A tennis ball traveling at 10.0 m/s is returned by Venus Williams. It leaves her racket with a speed of 36.0 m/s in the opposite direction from which it came. a) What is the change in momentum of the tennis ball? b) If the 0.060-kg ball is in contact with the racket for 0.020 s, with what average force has Venus hit the ball?

Solution: In this exercise, the tennis ball is coming toward Venus with a speed of 10.0 m/s in one direction, but she hits it back with a speed of 36.0 m/s in the opposite direction. Therefore, you must think about velocity vectors and call one direction positive and the opposite direction negative.

a. *Given:* $v_o = -10.0$ m/s
$v_f = 36.0$ m/s
$m = 0.060$ kg

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v = m(v_f - v_o)$

Solve: $\Delta p = m(v_f - v_o) = (0.060 \text{ kg})[36.0 \text{ m/s} - (-10.0 \text{ m/s})] = \mathbf{2.8\ kg \cdot m/s}$

b. *Given:* $m = 0.060$ kg
$\Delta v = 46.0$ m/s
$\Delta t = 0.020$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.060 \text{ kg})(46.0 \text{ m/s})}{(0.020 \text{ s})} = \mathbf{140\ N}$

Example 4: To demonstrate his new high-speed camera, Flash performs an experiment in the physics lab in which he shoots a pellet gun at a pumpkin to record the moment of impact on film. The 1.0-g pellet travels at 100. m/s until it embeds itself 2.0 cm into the pumpkin. What average force does the pumpkin exert to stop the pellet?

Solution: First, convert g to kg and cm to m.

$$1.0 \text{ g} = 0.0010 \text{ kg} \qquad 2.0 \text{ cm} = 0.020 \text{ m}$$

Before you can solve for the force in the exercise, you must first know how long the force is being exerted. Remember, in order to find the time, you must use the average velocity, v_{av}.

$$v_{\mathrm{av}} = \frac{v_{\mathrm{f}} + v_{\mathrm{o}}}{2} = \frac{0 \text{ m/s} + 100. \text{ m/s}}{2} = 50.0 \text{ m/s}$$

Given: $v = 50.0$ m/s, $\Delta d = 0.020$ m

Unknown: $\Delta t = ?$
Original equation: $\Delta d = v\Delta t$

Solve: $\Delta t = \dfrac{\Delta d}{v} = \dfrac{0.020 \text{ m}}{50.0 \text{ m/s}} = 0.00040 \text{ s}$

Now we can solve for the force the pumpkin exerts to stop the pellet.

Given: $m = 0.0010$ kg, $\Delta v = 100.$ m/s, $\Delta t = 0.0040$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.0010 \text{ kg})(100. \text{ m/s})}{(0.00040 \text{ s})} = \mathbf{250\ N}$

Practice Exercises

Exercise 1: On April 15, 1912, the luxury cruiseliner *Titanic* sank after running into an iceberg. a) What momentum would the 4.23×10^8-kg ship have imparted to the iceberg if it had hit the iceberg head-on with a speed of 11.6 m/s? (Actually, the impact was a glancing blow.) b) If the captain of the ship had seen the iceberg a kilometer ahead and had tried to slow down, why would this have been a futile effort? (Read more about the *Titanic* at http://www.titanic.com)

a) $p = mv = (4.23 \times 10^8 \text{ kg})(11.6 \text{ m/s}) = \mathbf{4.91 \times 10^9\ kg \cdot m/s}$
b) The ship's momentum is so great that the ship is very difficult to stop.

Answer: **a.** **4.91×10^9 kg·m/s**

Answer: **b.** **see above**

Exercise 2: Auto companies frequently test the safety of automobiles by putting them through crash tests to observe the integrity of the passenger compartment. If a 1000.-kg car is sent toward a cement wall with a speed of 14 m/s and the impact brings it to a stop in 8.00×10^{-2} s, with what average force is it brought to rest?

$F = m\Delta v/\Delta t$ = (1000. kg)(14.0 m/s)/(8.00×10^{-2} s) = **175,000 N**

Answer: **175,000 N**

Exercise 3: Rhonda, who has a mass of 60.0 kg, is riding at 25.0 m/s in her sports car when she must suddenly slam on the brakes to avoid hitting a dog crossing the road. She is wearing her seatbelt, which brings her body to a stop in 0.400 s. a) What average force did the seatbelt exert on her? b) If she had not been wearing her seatbelt, and the windshield had stopped her head in 1.0×10^{-3} s, what average force would the windshield have exerted on her? c) How many times greater is the stopping force of the windshield than the seatbelt?

a) $F = m\Delta v/\Delta t$ = (60.0 kg)(25.0 m/s)/(0.400 s) = **3750 N**
b) $F = m\Delta v/\Delta t$ = (60.0 kg)(25 m/s)/(1.0×10^{-3} s) = **1.5×10^6 N**
c) (1.5×10^6 N)/(3750 N) = **400 times as great**

Answer: **a.** **3750 N**

Answer: **b.** **1.5×10^6 N**

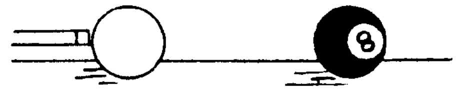
Answer: **c.** **400 times as great**

Exercise 4: On October 17, 2006, the population of the United States reached 300. million. If all of those people jumped up in the air simultaneously, pushing Earth with an average force of 800. N each for a time of 0.10 s, what would happen to the 5.98×10^{24} kg Earth? Show a calculation that justifies your answer.

$\Delta v = F\Delta t/m$ = ($300. \times 10^6$)(800. N)(0.10 s)/(5.98×10^{24} kg) = **4.0×10^{-15} m/s**
Therefore, Earth's motion would not be measurable.

Answer: **4.0×10^{-15} m/s**

Exercise 5: In Sharkey's Billiard Academy, Maurice is waiting to make his last shot. He notices that the cue ball is lined up for a perfect head-on collision, as shown. Each of the balls has a mass of 0.0800 kg and the cue ball comes to a complete stop upon making contact with the 8 ball. Suppose Maurice hits the cue ball by exerting a force of 180. N for 5.00×10^{-3} s, and it knocks head-on into the 8 ball. Calculate the resulting velocity of the 8 ball.



$\Delta v = F\Delta t/m = (180.\ \text{N})(5.00 \times 10^{-3}\ \text{s})/(0.0800\ \text{kg}) =$ **11.3 m/s**
Since the balls each have the same mass, the second ball acquires a velocity of 11.3 m/s and the first ball comes to rest.

Answer: **11.3 m/s**

Exercise 6: During an autumn storm, a 0.012-kg hail stone traveling at 20.0 m/s made a 0.20-cm-deep dent in the hood of Darnell's new car. What average force did the car exert to stop the damaging hail stone?

$\Delta t = \Delta d/v = (0.0020\ \text{m})/(10.0\ \text{m/s}) = 2.0 \times 10^{-4}\ \text{s}$
$F = m\Delta v/\Delta t = (0.012\ \text{kg})(20.0\ \text{m/s})/(2.0 \times 10^{-4}\ \text{s}) =$ **1200 N**

Answer: **1200 N**